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E.G.Bessonov

To Foundations of Classical Electrodynamics

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Abstract

In the present work a number of questions concerning foundations of the classical electrodynamics are discussed. First of all these are the law of conservation of energy and the introduction of particles in classical electrodynamics. We pay attention to a logical error which appears at the interpretation of the Poynting's theorem ostensibly following from the equations of Maxwell-Lorentz as the law of conservation of energy. It was shown that the laws of conservation of energy and momentum of the system of electromagnetic field and charged particles does not follow from the equations of electrodynamics. The violation of these laws is displayed when the energy of particles is changed. Particular examples are considered which make it possible to restrict a possible form of fields of a non-electromagnetic origin. Thus we transfer the essence of difficulties of electrodynamics deeper in its foundations. We hope that this work will permit to produce a more comprehensive analysis and to stimulate the further development of the foundations of the classical and quantum electrodynamics.

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Contents

1 Introduction

Usually speaking about difficulties arising in classical electrodynamics, for example, at introduction in it particles, establishment of the equation of motion of particles in view of radiation reaction forth and other it is emphasized that these difficulties are non-principle, that classical electrodynamics is the consistent relativistic theory. In addition everybody refer to the laws of conservation of energy, linear and angular momentum for particles and fields.

In the present work we show that similar references on the conservation laws are incorrect. We pay attention to a typical logical error in textbooks on classical electrodynamics when the law of conservation of energy (Pointing's theorem) is proved for the case of a system consisting of electromagnetic field and charged particles. The violation of this law is displayed when the energy of particles is changed. Private examples are considered which allow to impose certain restrictions on a possible form of fields of non-electromagnetic origin. Thus we transfer the essence of difficulties of electrodynamics deeper in its foundations.

2 The laws of conservation in electrodynamics

Below we shall present the typical proof of the law of conservation of energy in electrodynamics of Maxwell-Lorentz for a system consisting of an electromagnetic field and charged particles to specify then a logic error. There are three basic moments in the proof: [1, p.106]-[4].

1. From Maxwell equations an equation follows

$$\frac{\partial}{\partial t} \int \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} dV + \int \vec{j}\vec{E}dV = 0, \tag{1}$$

where $\vec{E}$, $\vec{H}$ are vectors of an electric and magnetic field strengths, $\vec{j} = \rho\vec{v}$ vector of a current density, ρ a particle charge density, $\vec{v}$ vector of velocity of a motion of the given element of volume of a charge of the particle and the integration is made over the whole space.

2. From a relativistic equation of motion of a charged particle and definition of work the law of energy change of a particle is derived

$$\frac{d\varepsilon}{dt} = \int \rho\vec{E}\vec{v}dV. \tag{2}$$

It is considered that the energy of a particle is the value

$$\varepsilon = \frac{mc^2}{\sqrt{1 - v^2/c^2}}, \tag{3}$$

where m is a mass of a particle, v a velocity of a particle, c a velocity of light. The dependence of the particle's energy on velocity is determined by the requirements of the special theory of relativity. At introduction of particles in electrodynamics it is usually postulated, that the energy of a particle consists partially on the energy of it's own electromagnetic field connected with a particle ε^{em}, and partially from the energy of it's field of a non-electromagnetic origin $\varepsilon^{not\,em}$ ($\varepsilon = \varepsilon^{em} + \varepsilon^{not\,em}$).

3. It is postulated that the value

$$W = \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} \tag{4}$$

is the density of the energy of an electromagnetic field.

4. By elimination of the value $\int \vec{j}\vec{E}dV = \int \rho\vec{v}\vec{E}dV$ from the equations (1) and (2) we will obtain the expression $(d/dt)\varepsilon_\Sigma = 0$. It follows from this equation that the value

$$\varepsilon_\Sigma = \int \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} dV + \sum_i \varepsilon_i = const, \tag{5}$$

where ε_i is the energy of a particle with an index i, ε_Σ the total energy of a system which is a sum of energies of an electromagnetic field and particles in the hole space. The expression (5) is treated as the law of conservation of energy in electrodynamics.

Vectors $\vec{E}$, $\vec{H}$ in (5) represent a total electromagnetic field created by a system of particles. Volume integral of the energy density of the electromagnetic field represents a system of a self-energy (inertial energy) of particles, an electromagnetic interaction energy of particles (potential energy) and an energy of free fields. In a general case these components of energy are not separated. The radiation field can be selected only in a wave zone.

The error in the presented proof consist in unification of physically inconsistent equations (1) - (4) in one system. This inconsistency occurs because of non-consistent introduction of particles in electrodynamics. Namely, if we start from the Maxwell equations and postulate (4) then we should adopt that the energy of a charged particle ε_i in (5) partially consists of an energy of own electromagnetic field created by it. But the energy of a particle is included in the first term of this equation[1]. It means that the energy of own electromagnetic field of a particle is contained in the equation (5) twice. Just this fact leads to the logic error in the proof of the law of conservation of energy and because of which the equation (5) conflicts with the initial equations. It is not difficult to understand the reasons according to which this error was not exposed for a so prolonged time.

1. Usually the relativistic expressions of energy, momentum and mass of particles are introduced within the framework of the special theory of relativity. From this theory follows that the energy and momentum of particles should have certain transformation properties regardless to a nature of an origin of these values. Mass acts in a role of coefficient of proportionality between vectors of a momentum and velocity of particles.

2. In the electrodynamics the received values of energy and momentum of particles are introduced through the equations of a motion. The joint solution of the equations of motion for fields and particles reduces to the expression (5), which is treated as the law of conservation of energy. At this it is postulated that the value (4) is a density of the energy of an electromagnetic field.

3. The nature of mass, energy and momentum of particles are discussed later after the electrical and magnetic fields created by a uniformly moving spherically symmetrical particle and connected with these fields the energy and momentum of the particle of the electromagnetic origin were calculated. It turned out that these values have not correct transformation properties following from the special theory of relativity[2]. For this purpose and with the purpose of holding of a particle charge in equilibrium it is necessary to introduce attraction fields of non-electromagnetic origin the energy and momentum of which have wrong transformation properties of the other form. It is postulated that in a sum the fields of the electromagnetic and non-electromagnetic origin are reduced to experimentally observable values of energy and momentum of particles[3].

If after these remarks we return to the expression (5) then it is not difficult to notice that the energy of particles of an electromagnetic origin is introduced in this expression twice, and consequently the standard treatment of this expression is incorrect.

Unfortunately the laws of conservation, as a rule, were proved only to emphasize the consistency of the electrodynamics its perfection and in that form in which they were received they did not usually used since on the basis of only the laws of conservation it is possible to solve a small number of simple problems not representing practical interest[4]. Therefore the error in the proof, which would

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[1]It remains in a case of one uniformly moving particle.

[2]Besides a correct factor $1/\sqrt{1 - v^2/c^2}$ there are the incorrect factors $(1 + v^2/3c^2)$ in energy and $(4/3)$ in the momentum of particles[2].

[3]In the case of a non-uniform motion the concepts of energy and momentum of the electromagnetic origin were not discussed. Non-obviously they are taken equal to the appropriate values for the particles moving uniformly with the same velocity.

[4]It is possible to point out only on close questions connected, for example, with renormalization of mass in a classical electrodynamics (see [1], § 65, Problem 1, where in the expression for a sum of

be possible to establish by a comparison of the solutions following from the laws of conservation and from the equations of a motion, on particular examples was not detected. The next example illustrate this conclusion.

2.1   Example 1

There is an immovable spherically symmetrical charge q. On a distance a a particle with a charge e and mass m is located. In some moment the particle start to move and, being accelerated, leaves to infinity. Compare the energy of a particle calculated from the law of the energy conservation (5) and from the equations of motion.

Let us write down the expression (5) for an initial and a final states of the system and equate received expressions. Then we shall determine the kinetic energy of a particle

$$T = mc^2(\gamma - 1) = \frac{eq}{a} - \varepsilon_{rad}^{em} - (\varepsilon_e^{em} - \varepsilon_{e0}^{em}), \tag{6}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$, eq/a is the initial potential energy of the particle, ε_{rad}^{em} the energy of an electromagnetic field radiated by the particle, ε_e^{em} and ε_{e0}^{em} the energy of the own electromagnetic field of the moving particle and the particle at rest accordingly.

Calculation of a kinetic energy of a particle by the solution of the equation of a motion of the particle in a given field of the charge q will lead to an expression

$$T = \frac{eq}{a} - \varepsilon_{rad}^{em}. \tag{7}$$

As was to be expected, an extra term in (6) is equal to a difference between own energies of electromagnetic fields of accelerated and motionless particles. For the spherically symmetric distribution of particle's charge the value $\varepsilon_e^{em} - \varepsilon_{e0}^{em} = \varepsilon_{e0}^{em}[\gamma(1 + v^2/3c^2) - 1]$ [2].

2.2   Example 2

Investigate the transformation properties of the energy ε_Σ determined by the expression (5).

For the case of one uniformly moving particle the value ε_Σ depends on velocity by the law which differ from (3) as it has not correct transformation properties since this properties has the second but has not the first term of expression (5) which is equal to the electromagnetic self-energy of the particle [2]. The similar statement is valid for the beam of charged particles where the situation is intensified in addition by the circumstance that besides the electromagnetic self-energy of particles the first term in (5) will include the electromagnetic energy of interactions of particles which have incorrect transformation properties as well.

Thus transformation properties of the energy ε_Σ do not satisfy to the requirements of the special theory of relativity.

2.3   Remarks

1. In the case of pointlike particles the first term in (5) and consequently the value ε_Σ are infinite. They are changed to the infinitely large value when the particle's velocity change is finite. Since in the process of the evolution of a system the velocity of particles can be simultaneously increased or decreased then contrary to (5) in this case $\varepsilon_\Sigma \neq const$. On the other hand for the pointlike particles the proof of the laws of conservation operating with indefinitely large values cannot be correct.

energies of particles and fields created by them and written down with accuracy of up to the terms of the second order of v/c without the analysis of legitimacy the energy of particles of an electromagnetic origin is rejected).

It is interesting to know why did not anybody pay attention to this moment till now? For example, in the textbook [1, p.266] the law of conservation of energy is proved for pointlike particles, but in the process of the proof there is no scientific substantiation of the correctness of the mathematical operations with the indefinitely large values. At the same time later discussing the difficulties which appear in the electrodynamics when the runaway solutions appear we can read in this textbook: "A question can arise. How the electrodynamics satisfying to the law of conservation of energy can lead to an absurd result when a free particle increases its energy by unlimited way. The roots of this difficulties are, actually, in the mentioned earlier infinite electromagnetic "mass" of elementary particles. When we write finite mass of a charge in the equations of motion we, as a matter of fact, add to it formally an infinite negative "mass" of non-electromagnetic origin which together with an electromagnetic mass would result in final mass of a particle. Since, however, the subtraction of one from another of two infinities is not quite correct mathematical operation then it lead to a number of further difficulties including to the specified one here".

The logic however suggests that it is necessary the reasoning about non-correctness of mathematical operations to transfer to the proof of the law of conservation of energy instead of to base on this law.

2. In the electrodynamics of Maxwell-Lorentz there could not be a model of particles with pure electromagnetic nature of mass. Differently all energy would have electromagnetic nature and the second term in (5) should be absent. On the other hand the energy of charged particle ε cannot have pure non-electromagnetic nature since in a case, for example, of one particle the energy of extraneous forces applied to a particle will be transformed not only to the value ε and to the energy of the emitted radiation but also to the electromagnetic self-energy ε^{em} of the particle.

From the equations of electrodynamics instead of (5) a conclusion about conservation of a sum of energies of electromagnetic and non-electromagnetic origin possessing the correct transformation properties should follow. From these energies on certain unknown for the present principles it could be possible to select the energy of particles.

3. When the equations of motion and (2) are solved then only an external and a part of an electromagnetic self-field of a particle corresponding to a radiation reaction forth are taken into account. At the same time the inertial field of the particle is rejected[5] since it is considered that the fields of the non-electromagnetic origin produce a field of force equal to it by value but with the opposite sign[6]. Such "volitional" renormalization of a mass in the equations of motion of particles and also the rejection of the terms in the individual examples similar to the last term in (6) contradicts to a general principles which are guided at a derivation of (5).

In avoidance of misunderstanding we shall emphasize that the mentioned inconsistency of the equations takes place only in the electrodynamics of fields and particles and is displayed at the energy change of particles. For example, in the case of macroscopic electrodynamics the inconsistency of the introduced equations with the equations of the electromagnetic field does not appear.

So if the material bodies are motionless then it is possible to write down the current density in them as

$$\vec{j} = \lambda(\vec{E} + \vec{E}^{extra}), \tag{8}$$

where λ is a factor the electrical conductivity, $\vec{E}^{extra}$ the vector of the electric field strength of the extraneous forces.

In this case the expression (1) can be presented as

$$\frac{\partial}{\partial t} \int \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} dV = \int \vec{j}\vec{E}^{extra} dV - \int \frac{|\vec{j}|^2}{\lambda} dV, \tag{9}$$

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[5]In a non-relativistic case the electric field strength corresponding to the radiation reaction forth is proportional to the derivative of the particle acceleration and the electric field strength corresponding to the inertial field is proportional to the particle acceleration.

[6]In case of the pointlike particles the inertial fields, theirs energies and the electromagnetic masses are infinitely large values.

according to which it follows that the increment of the energy of an electromagnetic field is equal to an excess of the work of the extraneous electromotive forces over the heat generation. This is the law of the conservation of energy in electrodynamics including the motionless bodies [5].

In this case the proof is strict since the energy of the material bodies of an electromagnetic origin is absent or in the case of charged bodies is constant.

3 Introduction of particles in electrodynamics

We have shown, that expression (5), does not describe the law of conservation of energy in electrodynamics of Maxwell-Lorentz. In the present section particular examples are considered allowing directly i.e. without basing on expression (5) to illustrate inconsistency of the equations of motion of particles and field in electrodynamics of Maxwell-Lorentz or, at least, to impose certain restrictions on a possible type of fields of non-electromagnetic origins. In this case we shall proceed from the existence of the law of conservation energy for the electromagnetic field and particles in unknown for the present form which is differ from (5).

3.1 Example 3

Two identical charged particles are brought closer to each other with equal constant velocities by extraneous forces along an axis x up to a distance a_1. Then the extraneous forces are switched off and particles being decelerated continue to be brought closer by inertia until they will stop on a distance $a_2 < a_1$. After the stop of particles the extraneous forces are switched on again to keep particles in the state of rest.

Show that contrary to the law of conservation of energy[7] the energy of the considered system in the final state is higher than the energy spent by extraneous forces on acceleration of particles and their further bringing closer.

The idea of this example is the next. The electrical field of a uniformly moving particle is flattened in the direction of motion such a way that on the axis of motion at a distance a its value $E_\parallel$ is γ^2 times less then the electrostatic field of a charge at rest being at the same distance from the observation point [1]-[4]

$$E_\parallel^{mov} = \frac{1}{\gamma^2}E_\parallel^{rest} = \frac{e^2}{\gamma^2 a^2} \tag{10}$$

To transform to particles the initial velocity and bringing closer them up to a distance a_1 the extraneous forces must transform them the energy

$$\varepsilon_1 = 2mc^2(\gamma - 1) + \frac{e^2}{\gamma^2 a_1} + \varepsilon_{1rad}^{em}, \tag{11}$$

where ε_{rad}^{em} is the energy radiated by particles in the process of acceleration.

The energy of system in the final state is equal

$$\varepsilon_2 = \frac{e^2}{a_2} + \varepsilon_{1rad}^{em} + \varepsilon_{2rad}^{em}, \tag{12}$$

where ε_{2rad}^{em} is the energy of radiation emitted by particles at the process of deceleration.

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[7]As the repulsive forces between moving particles $eE_\parallel^{mov}$ are weakened γ^2 times in comparison with a static case then on a principle of bringing closer of particles to each other with relativistic velocities and subsequent separation them under non-relativistic velocities one could construct the perpetum mobile.

The value

$$\varepsilon_2 - \varepsilon_1 = \frac{e^2}{a_2} - \frac{e^2}{\gamma^2 a_1} + \varepsilon_{2rad}^{em} - 2mc^2(\gamma - 1), \tag{13}$$

which is equal to a difference between the energy transformed by charged particles to extraneous forces at separation of these particles and energy transformed by extraneous forces to particles to bring closer the particles.

It follows from (13) and conditions $\varepsilon_{2rad}^{em} > 0$, $a_2 < a_1$ that in this case there is an excess in energy $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1 > 0$ which is differ from zero at least at

$$A_1 < r_e \frac{\gamma + 1}{2\gamma^2}, \tag{14}$$

where $r_e = e^2/mc^2$ is the classical radius of a particle.

The function $\Delta\varepsilon(a_1)$ is obviously analytic one[8]. It means that $\Delta\varepsilon \neq 0$ at arbitrary a_1.

In this case if we start from the low of conservation of energy of the unknown for the present form which is differ from (5) then we are forced to impose restrictions on the range of applicability of the Maxwell equations and in particular on the Coulomb's low at small distances $a \leq r_e$. It is possible also to conclude that the fields of a non-electromagnetic origin can not be short-range one. After a stop of particles these fields lead to an attraction between them on distances $a \leq r_e$ with forces comparable with electrostatic repulsive forces. It means that in this case the classical electrodynamics cannot be built without the account of forces of a non-electromagnetic origin or that it is necessary all at once to build the unified theory of fields where the fields of non-electromagnetic origin on the same level with electromagnetic fields will be described by definite equations[9].

It is possible to assume also, that the size of particles should be comparable with the size r_e. Then the position of particles can not be determined by three coordinates and at the description of the particle's motion which draw closer to one another up to a distance $a \sim r_e$ it is necessary to take into account their internal structure.

3.2   Example 4

Show that the energy of an electromagnetic field radiated by a particle moving along a line of forces of a homogeneous electrical field of a capacitor under definite value of the field strength can be higher than the energy transformed by the field of the capacitor to the particle and find the value of the field.

The energy of an electromagnetic field radiated by a particle is equal

$$\varepsilon_{rad}^{em} = \frac{2e^4 E^2}{3m^2 c^3}\Delta t, \tag{15}$$

where $\Delta t = (cm/eE)\sqrt{(eEl/mc^2 + 1)^2 - 1}$ is the time of the particle motion in the capacitor of the length l [1-4].

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[8]Remind that if two analytical functions coincide within some range of variables then they are identically equal through the whole range of these functions.

[9]It is possible to accept that in the equations of motion of charged particles and further in expressions (2),(5) a mass m_i and an energy ε_i have a non-electromagnetic origin. In this case it would be possible to interpret the inertial coefficient automatically arising in the equation of a motion of particles as a mass of an electromagnetic origin and the energy of particles in (5) would be composed from a part of an energy of electromagnetic origins (first term) and from an energy ε_i of non-electromagnetic origin. However such approach would be an acceptable way of introduction of fields of non-electromagnetic origin in the electrodynamics of Maxwell-Lorentz (the way based on a postulated law of conservation of energy) but would not be the proof of existence of this law in the framework of the electrodynamics.

The energy transferred by the electrical field to the particle is equal

$$\Delta\varepsilon = eEl. \tag{16}$$

It should follow from the law of conservation of energy that this work is equal to a sum of the emitted energy ε_{rad}^{em} and the kinetic energy of a particle $mc^2(\gamma - 1)$ or

$$\Delta\varepsilon = \varepsilon_{rad}^{em} + mc^2(\gamma - 1). \tag{17}$$

But the relation (17) cannot be valid at any fields E in view of the fact that $\Delta t > l/c$, $mc^2(\gamma - 1) > 0$, $\varepsilon_{rad}^{em} \sim E^2$, $\Delta\varepsilon \sim E$. It is violated at the fields

$$E > \frac{3e}{r_e^2}. \tag{18}$$

The energy of the electromagnetic radiation emitted by the particle in the capacitor at such field strengths will be higher than the energy that one needs to move it from one capacitor plate to another. This result contradicts to the law of the conservation of energy.

It is possible to eliminate this contradiction if to assume that particles in the classical electrodynamics have the final dimensions. In this case in (15) there will appear the coherence factor of the radiation which is decreased when the value E is increased. To assume that the radiation reaction forth appear in the homogeneous electrical field for charged particles, which is increased with increasing of the value E is impossible without the conflict with the equations of motion [1],[6].

4 Conclusion

In electrodynamics there are many "open" or "perpetual" problems such as the problem of the self-energy and momentum of particles, the nature of the particle's mass, the problem of the runaway solutions. There is a spectrum of opinions concerning the importance and the ways of a finding of the answers on these questions. A number of these opinions we shall give below.

Unfortunately the efforts of the majority of the authors are directed more often to avoid similar questions than to solve them. In addition they base themselves on the laws of conservation ostensibly following from the electrodynamics in the most general case and presenting electrodinamics as the consistent theory. In such stating the arising questions do not have a physical subject of principle and the difficulties in their solution are on the whole in the field of the mathematicians.

It is shown in the present work that a relation (5) does not express the law of conservation of energy in electrodynamics of Maxwell-Lorentz. The error in the treatment of this expression is the consequence of insufficiently precise definitions of the basic concepts of the theory and its logically inconsistent construction. It is shown also that this electrodynamics is incompatible with the concept of pointlike particles even in that case, when a correct transformation properties are attached to them.

It follows, in particular, that in the process of any discussion of the existing difficulties of the theories it is impossible to refer on the law of conservation of energy in electrodynamics in the form, which was done, for example, in the textbooks [1],[6],[7].

We hope that this work will permit to produce a more comprehensive analysis and to stimulate the further development of the foundations of the classical and quantum electrodynamics.

5 Appendix

The electromagnetic field in vacuum is described by the Maxwell equations

$$rot\vec{E} = -\frac{\partial \vec{H}}{\partial t}, \tag{19}$$

$$div\vec{H} = 0, \tag{20}$$

$$rot\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{21}$$

$$div\vec{E} = 4\pi\rho \tag{22}$$

The typical proof of the law of conservation of energy in electrodynamics is derived according the following scheme [1].

Let us multiply both parts of the equation (19) on $\vec{H}$ and both parts of the equation (21) on $\vec{E}$ and subtract the received equations term by term

$$\frac{1}{c}(\vec{E}\frac{\partial \vec{E}}{\partial t}) + \frac{1}{c}(\vec{H}\frac{\partial \vec{H}}{\partial t}) = -\frac{4\pi}{c}\vec{j} - (\vec{H}rot\vec{E} - \vec{E}rot\vec{H}). \tag{23}$$

Using the known formula of the vector analysis $div[\vec{a}\vec{b}] = \vec{b}\,rot\,\vec{a} - \vec{a}\,rot\,\vec{b}$ we rewrite this relation in the form

$$\frac{\partial}{\partial t}\frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} = -\vec{j}\vec{E} - div\vec{S}, \tag{24}$$

where $\vec{S} = (c/4\pi)[\vec{E}\vec{H}]$ is the Pointing vector.

Let us integrate (24) through some volume and apply the Gauss theorem to the second term from the right. Then we shall receive the equation

$$\int(\frac{\partial}{\partial t}\frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} + \vec{j}\vec{E})dV = -\oint \vec{S}d\vec{f}, \tag{25}$$

where $d\vec{f} = \vec{n}df$ is a vector of an element of a surface determined by the area df and unit vector $\vec{n}$ normal to the surface and directed outside of the closed volume. If the system consists of charged particles then, according to (2), the integral $\int \vec{j}\vec{E}dV$ is written down in the form of a sum corresponding to all particles of system of a form $\sum e\vec{V_i}\vec{E}(\vec{r_i}) = \sum d\varepsilon_i/dt$. In this case (25) is transformed in

$$\frac{\partial}{\partial t}(\int \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi}dV + \sum_i \varepsilon_i) = -\oint \vec{S}d\vec{f}. \tag{26}$$

The value $\oint \vec{S}d\vec{f}$ is a flow of the energy of the electromagnetic field through a surface limiting the volume. If the integration is made through the total volume of the space i.e. if a surface of integration is withdrawn to infinity then the surface integral is disappeared (all fields and particles remain in space and do not go outside of the limits of the surface of integration) [1]. It follows the expression (5).

Notice that at the derivation of the expression (5) the expression (2) was used which is derived in the frameworks of relativistic mechanics independently on any introduction of the Maxwell equations and consequently concepts of the density of the energy of the electromagnetic field, energy and mass of particles of electromagnetic origins [1]. It is natural that in this case the mass and the energy of particles continue to be considered as some mechanical characteristics of particles and the fact is lost that with the charged particle the led self-fields are connected extending to the whole space, having an energy of an electromagnetic origin included in the first term of the expression (5), causing at acceleration the inertial and radiation reaction forces.

After introduction of the Maxwell equations and postulate (4) it would follow to postulate the law of conservation of energy for particles and field and being based on this base to search for ways of introduction of particles to classical electrodynamics just as it was done by Abraham and Lorentz [3] or to search the equations for fields of a non-electromagnetic origin and also the other ways of the solution of the problem.

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6 Spectrum of opinions

For some reason, the controversies which relate to the problems in understanding the basic principles of the theory of relativity and of quantum mechanics usually take a particularly acute form. they often "become personal," with mutual accusations of ignorance, etc.

E. L. Feinberg, Sov. Phys. Uspekhi, Vol. 18, No 8, (1976)

... In the field theory the authors come, for example, to a conclusion that "elementary particles cannot have of the final dimensions and should be considered as geometrical points." Such conclusion is hardly comprehended physically and philosophically and in any case requires physical and philosophical comment especially in the textbooks. Otherwise the reader of the textbook, a physicist or philosopher may do the conclusion that for Landau and Lifshitz "space is not the form of existence of matter" or that the elementary particles does not exist as there is nothing in a mathematical point. .....

Our duty is to achieve the state when in each general and special guide of physics - I mean in particular printed textbooks on physics - the philosophical base will find its clear expression. For the reader it is necessary the comprehensive organic penetration of correct philosophy into a specific matter.

S. I. Vavilov, Philosophskie voprosy sovremennoi physiki, Izdat. AN, SSSR, Moscow, 1952

Before it has become clear many theorists (Helmholtz, Hertz, Zommerfeld and other) have written multivolume, not to tell monstrous, works about electromagnetic self-energy of the hard electron .... Nowadays all these efforts seem to be vain; the quantum theory has replaced this point of view and now the tendency consists rather in avoiding of a problem of self-energy, than to solve it. But there will come a day when it will become central one again.

M.Born, Lecture read in Bern 16.7.1955 at an International conference devoted to 50 years of the theory of relativity (Naturwiss. Rundschaw), 1956 (from A. A.Typkin, Principle of relativity, Atomizdat, Moscow, 1975, p.23)

There is one strange moment in the previous consideration. Classical electrodynamics is a relativistic covariant theory. That is why it is possible to wait that correct calculation of any value the requirements of the Lorentz covariance will not be violated. Nevertheless in the Abraham-Lorentz model, apparently, such violation is available. The non-covariant electromagnetic part of self-energy and momentum of a charged particle is counterbalanced in this model by the non-covariant part made dependent on Poincare stresses such a way that the result has a relativistic covariance. Certainly it is possible to say that since for a stable state of a limited distribution of a charge the retained forces of a non-electromagnetic origin and corresponding fields are necessary then only total forces have a physical sense. A question is nevertheless natural - Is it possible to determine the purely electromagnetic part of self-energy and momentum such a way that it has the relativistic covariance? Such determination would have not only aesthetic value but would separate at least formally the question of stability from a question of Lorentz invariance.

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Dzh. Dzhekson, <i>Classicheskaya electrodynamica,</i> World, Moscow, 1965, p.651;
see also: J. D. Jackson, <i>Classical Electrodynamics,</i> John Wiley &. Sons, 1962
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... a completely satisfactory treatment of the reactive effects of radiation does not exist. The difficulties presented by this problem touch one of the most fundamental aspects of physics, the nature of an elementary particle. .... one might hope that the transition from classical to quantum-mechanical treatments would remove the difficulties. While there is still hope that this may eventually occur, the present quantum-mechanical discussions are best with even more elaborate troubles than classical ones. It is one of the triumphs of comparatively recent years ($\sim$1948-1950) that the concepts of Lorentz covariance and gauge invariance were exploited sufficiently cleverly to circumvent these difficulties in quantum electrodynamics. ... From a fundamental point of view, however, the difficulties still remain.

J. D. Jackson, <i>Classical Electrodynamics,</i> John Wiley &. Sons, 1975, p.781

In physics there are many "perpetual problems" the discussion of which contains for decades in the scientific literature let alone in textbooks.

The problem of radiation from a uniformly accelerated charge and most of other "perpetual problems" are undoubtedly of no major significance and this is precisely why they have remained insufficiently well explained for so long time. On the other hand, however, neglect of such methodical types of problems sometimes incurs vengeance.

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V. L.Ginzburg, Soviet Physics Uspekhi, v.12, No 4, p.565 (1970)
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<div align="center">
See also: R. P. Feynman, R. B. Leyghton, and M. Sands, The Feynman Lectures
on Physics, v.2, Addison-Wesley, Reading, Mass. (1963).
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